Cascading Failures Caused by Node Overloading in Complex Networks

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Joint Workshop on Cyber-Physical Security and Resilience in Smart Grids
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Motivation
Motivation

Definition (Cascading Failures)
A failure in a system of interconnected parts in which the failure of one part triggers the failure of successive parts.

- Phenomenon observed in power grids when one of the elements fails and shifts its load to nearby elements which may then become overloaded and fail
- Past research has focused on node and link failures
- This work introduces a new type of attack: Overload Attack
Abstraction for a Networked System

- Undirected graph $G = (V, E)$
- $V$ represents resources
- **Abstraction:** Two resources are linked by an edge if in case of failure of one resource, neighboring resources take over failed node’s task
- Each node bears at every moment in time a *load*
- Nodes have maximal manageable load, called the node’s *capacity*
- If the load of a node exceeds its capacity the node *fails* and its load is redistributed to its neighbors according to a *distribution rule*
Motivation

The Model

- A network \( N = (G, L, C, \omega) \) is a node and edge-weighted undirected graph \( G = (V, E) \) where
  - \( L : V \rightarrow \mathbb{R}^+ \) associates a load and
  - \( C : V \rightarrow \mathbb{R}^+ \) a capacity with each node
  - For each node \( v \) each neighbor \( u \) carries a positive weight \( \omega(v, u) \)

- Distribution rule:
  After failure of a non-isolated node \( v \) a non-failed neighbor \( w \) of \( v \) receives a part \( v \)'s load proportional to ratio of \( \omega(v, w) \) to sum of weights of all neighbors of \( v \)

\[
\Delta_v(w) = L(v) \frac{\omega(v, w)}{\sum_{u \in N(v)} \omega(v, u)}
\]
Example

![Diagram showing cascading failures caused by node overloading in complex networks.]

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**Attack Models**

- **Sequential attack**
  Failures of multiple network nodes occur at different times

- **Simultaneous attack**
  Multiple failures occur simultaneously

- Simultaneous attack model does not consider time domain in applying failures

- Even in sequential attack model concurrent failures can happen
  - If two neighboring nodes v and w fail at the same time, does v shed load to w before w fails or not?
  - Ambiguous definition of the cascading load distribution
Cascading Load Distribution

- $V_a$ current set of alive nodes
- $S \subseteq V_a$ set of nodes that fail at the same time
- Load distribution is an iterative process:
  - while $S \neq \emptyset$
    1. Load of nodes in $S$ is distributed to alive neighbors. This may increase load of some nodes beyond their capacity, i.e., they fail
    2. Determine the set $S$ of newly failed nodes
Overload Attack
Overload Attack

Definition (Overload Attack)

Attacker increases load of a node beyond its capacity.

Lemma

For each node $v \in V$ of a connected network $N$ there exits a value $L_{\text{crit}}$ such that an overload attack with load $L \geq L_{\text{crit}}$ on $v$ leads to failure of all nodes in $N$.

- Let $f_v(L)$ be number of nodes that fail when $v$ gets load $L$
- Observation: $f_v$ does not monotonically increase with $L$
- Reason: With increasing $L$ a path $p$ for redistributing load can get interrupted at some point. As a consequence some nodes receive considerable less load as before.
Overload Attacks
Research Problem

Problem

*How vulnerable are networks against increase of load of a small number of nodes given a limit for the relative increase of load of single nodes?*
Example: French Power Grid
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The Algorithm
Load Factor

Definition (Load Factor)

\[ \text{lf}(N) := \frac{1}{|V_a|} \sum_{v \in V_a} \frac{L(v)}{C(v)} \]

- The closer the load factor is to 1 the higher is the probability for a node failure to trigger a cascade of crashes.
- Strategy to weaken a network:
  Increase load of nodes as long as no load is lost. Note that load is lost if a node fails at a time all its neighbors have already failed.
Define a metric $p(v)$ that estimates potential of a node $v$ to raise load factor of a network.

For a node $v$ let $f_{\text{max}} C(v) + \epsilon \geq L_v > C(v)$ be the largest load increment of $v$ that does not lead to loss of load.

Alternatives for defining $p(v)$:
- Load factor after increasing load of $v$ by $L_v$
- Number of failed nodes due to increasing load of $v$ by $L_v$
- Additional load $L_v$

How to compute $L_v$?
- Iteratively increment load of $v$ by fixed value $inc$
Heuristic

- Repeat until all nodes have failed
  - Compute $p(v)$ for each node $v$
  - Select node $v_{\text{max}}$ with highest value for $p$
  - Increase load of $v_{\text{max}}$ to $L_{v_{\text{max}}}$
  - Compute remaining network

- Result: Sequence of nodes for sequential attack to force down all nodes
Application to French Power Grid
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Evaluation
Algorithm was applied to various networks (existing and artificial)

The following results are based on Western North American (WNA) power grid network with 4941 stations and 6594 transmission lines

Load:
- Degree of node

Capacity:
- Capacity of each node is first raised to satisfy the $N-1$ failure tolerance criterion
- Raised capacity is then scaled by a factor $T$ to raise robustness

Weights:
- Load-based and constant

Optimal solution for WNA grid is unknown!
Evaluation of the Metrics

maxLoad

maxFailure

maxLoadFactor
Influence of $f_{max}$
Number of Connected Components

maxFailure with $f_{max} = 1.0$

maxFailure with $f_{max} = 3.0$
Conclusion
Overload attack: New type of attack threatening the dynamical robustness of a complex network

Heuristic to select $k$ nodes such that sequential attack has a maximal impact on network while minimizing cost of attack

Evaluation for the WNA power grid network:
- By increasing the load of 0.5% of all nodes slightly above the capacity more than 50% of the network can be forced to fail

Similar results for scale-free networks (Barabási-Albert model)
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Initially all nodes have load $x$, the capacities are stated for each node. When node $v$ gets an additional load $x$ then it fails leading to the crash of all nodes. When node $v$ gets the additional load $x + 1$ then all nodes but $u_6$ and $u_7$ fail. In this case a considerable amount of load is lost at node $u_4$. All weights are equal to 1.